A Note on Labor Unions

Robert Trost
George Washington University

Julian Silk
George Washington University

March 21, 2003
A Note on Labor Unions

Robert Trost

Julian Silk

March 21, 2003

Abstract: A simple theory of labor unions is examined. This theory is appropriate for any supplier of intermediate goods or services with market power. The revenue expansion of labor (RX_l) is defined equal to the quantity of labor times marginal revenue product of labor. Quantities of union-supplied labor are set where the derivative of RX_l, or "marginal" revenue expansion MRX_l equals the supply function for labor. Some extensions and limitations of the theory are proposed.

Keywords: Labor unions, input monopoly, marginal revenue expansion, monopsony, elasticity, product quality

We thank John W. Ruser, Robert F. Phillips, Gale Boyd, Wooyoung Kim, Robert S. Goldfarb and David C. Ribar for discussions. All errors are our responsibility.
I. Introduction

Many microeconomics textbooks do not provide any analysis of labor unions (e.g., Mansfield/Yohe (2000)). If unions are monopolists in the market for labor, an economic theory of unions would seem to be an application of the microeconomic theory of monopoly. However, the standard model for existing unions, provided by Farber (1978) and Farber (1986), especially 1046-1052 and 1062-1063, is different. (Farber (1983) has a model for the establishment of unions.) The standard model takes unions to maximize a social utility function representing the decisions of their members, given the constraints of labor demand. (In Farber (1978), op. cit., this function is taken as the preferences of the median union member.)

Our object is to supply a modeling approach for unions acting as monopolists in the input market for labor. This approach may be consistent with one type of analysis of unions.\(^1\) Natural extensions are also proposed. Input monopoly models may help provide a testable alternative hypothesis to the standard model, and so clarify understanding of union behavior.

II. The Simple Model

Suppose there are firms supplying one output of uniform quality. Quantity supplied is \(q\). The output's price is \(p\).

To produce output, firms employ inputs, including labor. Let quantity of labor employed be \(l\). All firms share a production function, known to all market participants,

\[
y = f(l, i_1, \ldots, i_n) \tag{1}
\]

where \(i_1, \ldots, i_n\) are non-labor inputs used in production. Suppose this function is second-order differentiable in labor input.

In selling the output, the firms receive revenue

\[
R = p \cdot q \tag{2}
\]

Total costs incurred in production for each firm are

\[
TC = w \cdot l + \sum_{i=1}^{n} r_i \cdot i_i \tag{3}
\]

where \(w\) is the unit price of labor, and \(r_i\) are unit prices for each

---

\(^1\)This analysis has been provided informally by H. Gregg Lewis and John W. Ruser.
of the other inputs in production, and there are \( n \) other inputs.

\[
\text{MRP}_l = d(l) \tag{4}
\]

Profits are the difference between total revenues and total costs for the output. All firms maximize profits. Demand for labor is given by \( \text{MRP}_l \), the marginal revenue product of labor, which equals \( d(l) \), the demand for labor. Firms hire labor to the point of paying each laborer exactly their market value provided.

Two polar cases for labor demand exist. For perfect competition

\[
\text{MRP}_l = p \cdot MP_l = p \cdot \frac{\partial f}{\partial l} \tag{5}
\]

where \( MP_l \) is the marginal physical product of labor \( \frac{\partial f}{\partial l} \), the change in output with a unit change in labor (the first partial derivative of the production function with respect to labor).

For a monopolist in the output market, the demand for labor is

\[
\text{MRP}_l = MR \times MP_l = p \cdot (1 + (1/\varepsilon)) \cdot \frac{\partial f}{\partial l} \tag{6}
\]

where \( MR \) is marginal revenue for the a unit change in output, and \( \varepsilon < 0 \) is the price elasticity of demand in the output market.

Intermediate labor demands can be derived from intermediate market structures, e.g., Cournot oligopoly (Cowell (1986), 240-243).

The supply of labor (or average expenditure on labor per firm) is

\[
s(l) = g(w) \tag{7}
\]

where \( s(l) \) is the supply of labor and \( g(w) \) is upward-sloping (or at least nonnegative) in \( w \). (Backward-bending labor supply, e.g., Mansfield/Yohe, op. cit., 456, puts an upper limit on labor that is supplied to competitive markets with unchanging technology.)

Variable hours of work and labor-leisure utility maximization is sufficient for \( g(w) \) to be upward-sloping, but not necessary. Some laborers may be married and have children. High childcare costs, low wages and cost minimization can induce some laborers to exit the labor market to provide childcare (and other home production).

Labor market equilibrium is \( d(l) = s(l) \) (Mansfield/Yohe, op. cit., 460). Let the quantity of labor input this sets be \( l_c \), with corresponding wage \( w_c \). This situation is shown in Figure 1.
Now consider the labor union. Suppose firms can only hire union members (the labor force is 100% unionized). The union acts as a monopolist. The demand for its output, labor, from each firm is $\text{MRP}_l$. The union has marginal cost $s(l)$, the supply function for labor. Let union "revenue" be given by $RX_l = l \times \text{MRP}_l$, where $RX_l$ may be denoted the "revenue expansion" for labor. The derivative of the revenue expansion is $\text{MRX}_l$, and may be denoted "marginal revenue expansion for labor". The union then sets optimum earnings, by setting labor quantity where $\text{MRX}_l = s(l)$.

For the two polar cases described above,

$$\text{MRX}_l = p \times \text{MP}_l + p \times l \times \frac{\partial \text{MP}_l}{\partial l}$$

$$= \text{MRP}_l + p \times l \times \frac{\partial^2 f}{\partial l^2} \quad (8)$$

and

$$\text{MRX}_l = MR \times \text{MP}_l + l \times MR \times \frac{\partial \text{MP}_l}{\partial l}$$

$$= \text{MRP}_l + l \times p \times (1 + (1/\varepsilon)) \times \frac{\partial^2 f}{\partial l^2} \quad (9)$$

where $\frac{\partial^2 f}{\partial l^2}$ is the second partial derivative of the physical product of labor with respect to labor.

Intermediate cases in the output market (e.g., Cowell (1986), 240-243) generate intermediate cases for the labor union.

For the special case where $\text{MRP}_l = \alpha + \beta \times l$, the marginal revenue expansion for labor will have the simple form

$$\text{MRX}_l = \alpha + 2 \times \beta \times l \quad (10)$$
Call the quantity of labor for the labor union $l^*$. The union wage, $w^*$, is the marginal revenue product of this quantity. The union quantity is lower than the competitive quantity, and the union wage higher than the competitive wage. This is shown in Figure 2.

II. Monopsony

With a competitive supply of labor, a monopsony solution is well defined. Total expenditure for labor can be defined as

$$TE(l) = s(l) \cdot l$$  \hspace{1cm} (11)

Marginal expenditure for labor is the change in total expenditure for labor per unit increase of labor, or

$$ME(l) = \frac{dTE(l)}{dl} = s(l) + s'(l) \cdot l$$  \hspace{1cm} (12)

where $\frac{dTE(l)}{dl}$ is the derivative of total labor expenditure function, and $s'(l)$ is the derivative of the labor supply function.

The monopsony solution sets the quantity of labor where the marginal expenditure for labor equals the marginal revenue product for labor (Mansfield/Yohe, op. cit., 472), or

$$MRP_{-}l = s(l) + l \cdot s'(l)$$  \hspace{1cm} (13)

Call the monopsony quantity of labor $l_{-}m$, and the wage paid $w_{-}m$. The monopsony solution is shown in Figure 3.
It is often argued that the outcome is indeterminate if a monopsonist faces a labor union. Here the area of indeterminacy is an Edgeworth box-like figure area between the two solutions, in which bargaining takes place. (If both labor supply and labor demand are common knowledge to the union and the monopsonist, and labor supply and labor demand are both linear, the Edgeworth figure is a trapezoid.) The Edgeworth figure is shown in Figures 4 and 5.

A strike or a lockout reduces production to zero. But the range of production conditions over which intelligent bargaining settles is described by the Edgeworth figure.
In this case, the quantity of labor employed is greater with the labor union than the monopsony. The monopsony wage will always be less than the labor union wage, but the monopsony quantity of labor may be greater than the labor union quantity of labor. Rearranging terms for the monopsony solution,

\[ MRP \cdot l - l \cdot s'(l) = s(l) \]  \hspace{1cm} (14)

For monopolies, quantities are equal if \[ p \cdot (1 + \frac{1}{\varepsilon}) \cdot \frac{d^2 f}{dl^2} = -s'(l) \]. The labor union solution is the change in the product of quantity of labor times demand for labor equals the supply of labor. The monopsony solution is the change in the quantity of labor times the supply of labor equals the demand for labor. Cancelling the mutual quantity of labor in these expressions, the monopsony quantity of labor is less than the labor union quantity of labor if the supply function of labor is less elastic than is the demand function for labor, and vice versa. For linear functions \( MRP \cdot l = \alpha - \beta \cdot l \) and \( s(l) = \gamma + \delta \cdot l \), a simple condition holds for the labor union quantity
to be greater than the monopsony quantity, namely $\beta < \delta$.³

III. 2 Goods

Natural model extensions exist. For two goods, (1) changes to

$$R = p_1 q_1 + p_2 q_2$$

(15)

For production functions $MP_{11} = f_1(q_1, l, i_1, \ldots, i_n)$ and $MP_{21} = f_2(q_2, l, i_1, \ldots, i_n)$, the union rule is $MRX_{11} + MRX_{12} = s(l)$. For competitive firms, (8) changes to

$$MRX_{11} + MRX_{12} = MRP_{11} + p_1 l \frac{\partial f_1}{\partial l}$$

$$+ MRP_{21} + p_2 l \frac{\partial f_2}{\partial l} = s(l)$$

(16)

For monopoly, even if each good is produced independently, each good's sales may depend on the other's sales, i.e.,

$$MR = p_1 q_1 \left( \frac{\partial p_1}{\partial q_1} \right) + q_1 \left( \frac{\partial p_1}{\partial q_2} \right)$$

$$+ p_2 q_2 \left( \frac{\partial p_2}{\partial q_1} \right) + q_2 \left( \frac{\partial p_2}{\partial q_2} \right)$$

$$= p_1 \left( 1 + q_1 \frac{p_1}{p_1} \frac{\partial q_1}{\partial q_1} + q_2 \frac{p_1}{p_1} \frac{\partial q_2}{\partial q_2} \right)$$

$$+ p_2 \left( 1 + q_1 \frac{p_1}{p_2} \frac{\partial q_1}{\partial q_1} + q_2 \frac{p_2}{p_2} \frac{\partial q_2}{\partial q_2} \right)$$

(17)

These expressions can be put in terms of elasticities, namely,

$$MR = p_1 \left( 1 + \frac{1}{\varepsilon_1} \right) + p_1 \left( \frac{q_2}{p_1} \frac{\partial p_1}{\partial q_2} \right)$$

$$+ p_2 \left( 1 + \frac{1}{\varepsilon_2} \right) + p_2 \left( \frac{q_1}{p_2} \frac{\partial p_2}{\partial q_1} \right)$$

$$= p_1 \left( 1 + \frac{1}{\varepsilon_1} \right) + p_1 \left( \frac{1}{n_{21}} \right)$$

$$+ p_2 \left( 1 + \frac{1}{\varepsilon_2} \right) + p_2 \left( \frac{1}{n_{12}} \right)$$

(18)

³It is natural for the union and the monopsonist to try to create uncertainty about the Edgeworth figure in negotiation. The more the bilateral monopoly circumstances are repeated, the more this situation takes on the characteristics of a repeated game, and the smaller the area of the Edgeworth figure becomes. An optimizing union will recognize this. A case where a union performed suboptimally for its members is given by the discussion of the pressmen's union strike in Graham (1998), 509-576.)
\[ MR = p_1 \times (1 + \frac{1}{\varepsilon_1}) + (p_1 + p_2) \times (\frac{1}{n_{12}}) + p_2 \times (1 + \frac{1}{\varepsilon_2}) \]  

(19)

where \( \varepsilon_1 \) is the first good's own-price elasticity of demand, \( \varepsilon_2 \) is the second's own-price elasticity of demand, \( n_{12} \) is the first good's cross-price elasticity of demand for the second's price, and \( n_{21} \) is the second's cross-price elasticity of demand for the first's price. If these cross-price elasticities are symmetric, the labor union rule in the monopoly case is

\[ MRX_{L_1} = (p_1 \times (1 + \frac{1}{\varepsilon_1}) + p_2 \times (\frac{1}{n_{12}}) \times \frac{\partial f_1}{\partial l} \]
\[ + p_2 \times (1 + \frac{1}{\varepsilon_2}) + p_1 \times (\frac{1}{n_{12}}) \times \frac{\partial^2 f_1}{\partial l^2} \]
\[ + (p_2 \times (1 + \frac{1}{\varepsilon_2}) + p_1 \times (\frac{1}{n_{12}}) \times \frac{\partial f_2}{\partial l} + l \times p_1 \times \frac{\partial^2 f_2}{\partial l^2} + l \times p_2 \times \frac{\partial^2 f_2}{\partial l^2} = s(l) \]  

For perfect competition, where \( \varepsilon_1 = \varepsilon_2 = n_{12} = \infty \), this is

\[ MRX_{L_1} = p_1 \times \frac{\partial f_1}{\partial l} + l \times p_1 \times \frac{\partial^2 f_1}{\partial l^2} + p_2 \times \frac{\partial f_2}{\partial l} + l \times p_2 \times \frac{\partial^2 f_2}{\partial l^2} = s(l) \]  

(21)

The simplest production dependence has \( q_i = f_i(q_1, q_2, l, i_1, \ldots, i_n) \) and
\( q_2 = f_2(q_1, q_2, l, i_1, \ldots, i_n) \). The derivatives then become

\[
\frac{\partial f_1}{\partial l} = \frac{\partial f_1}{\partial q_1} \frac{\partial q_1}{\partial l} + \frac{\partial f_1}{\partial q_2} \frac{\partial q_2}{\partial l} \tag{22}
\]

\[
\frac{\partial f_2}{\partial l} = \frac{\partial f_2}{\partial q_1} \frac{\partial q_1}{\partial l} + \frac{\partial f_2}{\partial q_2} \frac{\partial q_2}{\partial l} \tag{23}
\]

\[
\frac{\partial^2 f_1}{\partial l^2} = \frac{\partial^2 f_1}{\partial q_1^2} \frac{\partial q_1}{\partial l} + \frac{\partial^2 f_1}{\partial q_2^2} \frac{\partial q_2}{\partial l} \frac{\partial q_2}{\partial l} + \frac{\partial f_1}{\partial q_1} \frac{\partial^2 q_2}{\partial l^2} + \frac{\partial f_1}{\partial q_2} \frac{\partial^2 q_2}{\partial l^2} \tag{24}
\]

\[
\frac{\partial^2 f_2}{\partial l^2} = \frac{\partial^2 f_2}{\partial q_1^2} \frac{\partial q_1}{\partial l} + \frac{\partial^2 f_2}{\partial q_2^2} \frac{\partial q_2}{\partial l} \frac{\partial q_2}{\partial l} + \frac{\partial f_2}{\partial q_1} \frac{\partial^2 q_2}{\partial l^2} + \frac{\partial f_2}{\partial q_2} \frac{\partial^2 q_2}{\partial l^2} \tag{25}
\]

Extension to many goods is obvious.

**IV. 2 Labor Supply Functions**

Suppose there are two deterministic labor supply functions (for labor of different races, genders, etc.). Equation (7) becomes:

\[
s_1(l) = g_1(w) \tag{26}
\]

and

\[
s_2(l) = g_2(w) \tag{27}
\]

Inverting these functions, quantities of labor are obtained as

\[
l_2 = s_2^{-1}(g_2(w)) \tag{28}
\]

and
\[ l_1 = s_1^{-1}(g_1(w)) \]  

(29)

Adding both sides of both equations together yields

\[ l = s_T^{-1}(G(w)) \]  

(30)

where \( s_T(.) \) is total labor supply. Inverting this equation yields

\[ s_T(l) = G(w) \]  

(31)

Extension to many types of labor is obvious.

The labor union solution is compared to the competitive solution in Figure 6. With the labor union, fewer of the higher cost laborers are employed than with perfect competition. But all who are employed receive higher wages than with perfect competition.\(^4\)

---

\(^4\) Let the number of higher cost workers with perfect competition be \( l_{-c2} \) and with the union be \( l_{-u2} \). (The numbers for lower cost workers are \( l_{-c1} \) and \( l_{-u1} \).) Note that \( l_{-c2} = s_2^{-1}(G(\text{w_c})) \), where \( \text{w_c} \) is the perfect competition wage. Also, \( l_{-u2} = s_2^{-1}(\text{MRX}_1(\text{l}^*)) \) where \( \text{l}^* \) is the total number of workers of both types hired with the union. Total income for the higher cost workers with perfect competition is \( G(\text{w_c}) \cdot l_{-c2} \). Total income for higher cost workers with the union is \( \text{MRP}_1((\text{l}^*)) \cdot l_{-u2} \). Because \( \text{MRP}_1((\text{l}^*)) \) is higher
Higher cost workers may not explicitly demand higher wages. Effective costs may depend on human capital and savvy.

Suppose there are valid records of durable goods sold in a market, but the records are difficult to access. Suppose the task is to learn the number of a competitor's durable goods outstanding in the market in 1 week, but a complete census requires 4 weeks. Savvy workers who know survey statistics can complete the task by providing a good estimate in 1 week. If the educated and savvy and those who aren't receive the same wage, the effective wage of those who aren't is 4 times the effective wage of those who are. Those not educated and savvy may well have employment involuntarily terminated, since they're not performing to requirements. A union acting as an input monopolist would set contract length based on information and changes in the market, given these types of workers. Such a union might solve for each period (e.g., a year) and discount based on market interest rates.

Contracts will not be negotiated by a process of direct democracy, but by the union's professional negotiating team. Elections will ratify contracts, but will enter into the negotiations directly only if information available to the union changes during voting, or if the union can't or doesn't make voting recommendations. Elections may also change union management (possibly separately from ongoing contract negotiation). In these cases, the model of Rosen (1986) may describe worker choice, and a median voter model (e.g., Downs (1957)) makes choice determinate.

A majority of low-effective-wage workers may not have employment

than \( G(w,c), \) income for higher cost workers will often be greater with the labor union than with perfect competition.

Employers see minority candidates lacking "soft skills" in language use (Moss and Tilly (2001)). This could be information sent per unit time in the context of employers and clients.

There is a social component for the educated: employers personally comfortable with the employee may tell the employee that surveys will be enough. Most educated will know they aren't performing well enough, even if they don't know how to do so.

Employers must be quick and adaptable to meet (possibly interproduct) competition (Jennings and Haughton (2001)).

Rosen's model has worker preferences \( U(C,D), \) where \( C \) is a market consumption good and \( D \) is a (dirty) consumption indicator of a job, Rosen, op. cit., 656. Here \( C \) might represent union teams that offer security (through cooperation), while \( D \) might be union teams that promise higher wages and additional benefits.
involuntarily terminated for any offered contract. Depending on seniority rules, their preferences may be decisive.\textsuperscript{8}

Union wages keep the \textit{same} higher cost workers employed, versus a more rapid turnover of higher cost workers in perfect competition. If the size of the peer group of higher cost workers and incentives are both sufficient, interaction between groups may increase high-cost labor supply enough to remove the group differences. A non-union case, Louisiana State University's success with African Americans chemists, is one example of these effects (Collins, Stanley, Warner and Watkins (2001)). Longer tenure may make teams more successful, with more productivity from both sets of workers (Hamilton, Nickerson and Owan (2001)). Positive union group interactions may thus increase contract length employers offer.

With union wages, firms substitute away from labor. Non-labor inputs are more intensively used. For non-separable production functions, labor cost rises \textit{at any wage}.\textsuperscript{9} If a new worker offers

\textsuperscript{8}Workers express preferences by choice of jobs with seniority for layoffs and promotion. Elderly workers may prefer long contract length to high wages, young workers the opposite. However, with seniority, these preferences may be reversed.

To improve quality often requires investment in new ideas and to consistently monitor output. To try their new ideas, entrants to a hierarchy often must wait for attrition, engage in protracted conflict, or to risk exit to another job or entrepreneurship.

\textsuperscript{9}As Spence (1973) notes, hiring is an investment decision. Varying capacity utilization by varying hours of incumbent workers does not risk added liabilities for a new hire for pensions, health insurance, tax, administrative cost, personal injury litigation, etc. If there is slack time, opportunity cost in lost revenues for management or incumbent worker experiments with new technology also falls. For competitive markets, net returns for a new hire must encompass all costs and alternatives for the firm to make the offer. Unions add negotiation costs.

Spence argues "As new market information comes in to the employer through hiring and subsequent observation of productive capabilities as they relate to signals, the employer's conditional probabilistic beliefs are adjusted, and a new round starts. The wage schedule facing the new entrants in the market generally differs from that facing the previous group.", \textit{op. cit.}, 359.

Not all employment has synchronized wage offers and hiring decisions. The sources of information are different. Data on revenues for the on-site worker arrive daily in normal business operations. Testing different offers adds administrative cost, takes time for results, shifts work focus from normal operations
labor at zero wage, managers must offer tasks, time and space to accept. Such management costs may preclude acceptance.  

V. Quality and Uncertainty

For uncertainty as totally exogenous states of the world, simple modifications suffice. Marginal revenue products are replaced by expected marginal revenue products or their expected utility, and wages by expected wages or their expected utility.

Parsons (1986) provides a starting point if quality and uncertainty interact. He assumes competitive employers and that all agents are risk-neutral. Value of output $V$ is given by $V = u + H + \theta$, where $u > 0$ is a parameter, $H$ is worker effort and $\theta$ is random error, $E[\theta] = 0$. Workers have utility function $U = U(w - R + H^2)$, with parameter $R$, and first and second derivatives $U' > 0$, $U'' < 0$. Workers set negative marginal utility $2R + H$ equal to expected marginal gain $u$. Optimal effort is $H^* = u/(2R)$. Wage equals expected marginal revenue product, or $w^* = u^*(H^*)$.

A unit improvement in quality brings a constant gain to the firm. If a unit increase in quantity for a fixed quality takes a constant

and increases the risk of legal/grievance claims (errors on offers increase with more frequent change in terms).

Suppose a year has an average of 251.2 work days. Counting vacation and sick leave, suppose this leaves 237 work days per year. Suppose 1 observation can be made per day for an onsite worker, and it can be decided whether to involuntarily terminate employment with 59-237 observations. Now suppose it takes 5 work days to make 1 observation whether offers at a constant real wage provide adequate labor supply from offsite. (Assume vacation and sick leave, etc. aren't relevant for observations on wage offers.) The same decision-making process takes 1.18-4.72 years.

Employer signals matter, too. An employee may be involuntarily terminated with the option of continued employment at a lower wage. But unemployment or severance pay may be based on the last pay rate received. Having been found defective, tenure at a minimal rate (without further option) may last only long enough to qualify the employee for unemployment/severance at the minimal rate. Union contracts may preclude such cherry-picking.

10Learning by doing (Arrow (1962), Gemery and Hogendorn (1993), Mishina (1999)) by incumbents substitutes for new hires with a wage of zero when quantity demanded is constant.
increase in inputs, technology has constant returns to scale.\footnote{Labor union behavior for constant returns to scale depends on firm entry and exit, as behavior does in a competitive market. Suppose labor supply is infinitely elastic at wage $w_0$, and production technology is $q = A \cdot l$. If optimal response to exit and entry is to increase labor income to match increases in firm revenue, $w = \frac{1}{2} \cdot p \cdot A$, where output price $p > w_0$.} If $\mu$ is price given quality, constant returns hold, and union wages are $\frac{1}{2} \cdot \mu$, then $H_u$, or union effort, will be $\mu/(4 \cdot R)$.

In general, suppose worker utility is $u(h, w)$, where $h$ remains worker effort and $w$ is wage. Suppose unit quality is $\zeta(h) + \theta$. As above, $\theta$ is random error, $E[\theta] = 0$. The quality production function is now $\zeta(.)$. It may not have constant returns to scale.\footnote{Krueger and Mas (2002) provide an example of a function for product quality that need not have constant returns to scale.}

Suppose expected marginal revenue products are $MRP_1(\zeta(h), l)$ and $MRP_h(\zeta(h), l)$ and agents are risk-neutral.\footnote{Risk aversion can be handled by considering the utility of marginal revenue products and marginal revenue expansions.} Revenue expansions are

\begin{equation}
RX_{-l} = l \cdot MRP_1(\zeta(h), l) \tag{32}
\end{equation}

and

\begin{equation}
RX_{-h} = h \cdot MRP_h(\zeta(h), l) \tag{33}
\end{equation}

Marginal revenue expansions then become

\begin{equation}
MRX_{-l} = l \cdot \frac{\partial MRP_1(\zeta(h), l)}{\partial l} + MRP_1(\zeta(h), l) \tag{34}
\end{equation}

and

\begin{equation}
MRX_{-h} = h \cdot \frac{\partial MRP_h(\zeta(h), l)}{\partial \zeta(h)} \cdot \frac{\partial \zeta(h)}{\partial h} + MRP_h(\zeta(h), l) \tag{35}
\end{equation}

The supply functions require further consideration. The (negative)
partial derivatives for the utility function \(-\frac{\partial u}{\partial l}\) and \(-\frac{\partial u}{\partial h}\) will form the left-hand sides of the supply function. To elicit distinct right-hand sides, the function \(G(w)\) will have to change to \(Z(w,x)\), a function of at least two independent variables.

Let \(w\) be one independent variable. Let \(x\) be a scalar or vector of variables that elicit labor effort (management monitoring, customer monitoring, good labor relations, interest of work, etc.).

For \(Z(w,x)\), the supply equations become

\[-\frac{\partial u}{\partial l} = \frac{\partial Z(w,x)}{\partial w}\]

(36)

and

\[-\frac{\partial u}{\partial h} = \frac{\partial Z(w,x)}{\partial x}\]

(37)

These partial derivatives may not be linear. For constant \(w\), \(Z(w,x)\) may be an "S-shaped" curve.

If the firm only operates with \(G(w)\), it fails to achieve even the union level of shirking. The union level is more than the competitive level if \(\frac{\partial MRP_h}{\partial \zeta(h)} \cdot \frac{\partial \zeta(h)}{\partial h} > 0\).

Clearly, there are cases where union-induced shirking completely determines quality. One is provided by Wesel (2001).

Low product quality may also be caused by poor raw materials, inefficient management or both. Deming (2000 (a), especially 327-330, and Deming (2000 (b)), especially ch. 7, shows quality variation from raw material inputs can outweigh shirking.\(^{15}\)

\(^{14}\)See Krueger and Mas, op. cit., and Hamilton, Nickerson and Owain op. cit., for the justification of the latter possibilities.

\(^{15}\)Deming (2000 (a), 79-80) shows that poor, scanty capital can install poor output quality, making labor efforts irrelevant.

Deming (2000 (b), 73-75) also argues that quality is optimal with monopoly. But perfectly competition may induce superior quality. For separable consumer response to price and quality, a simple monopoly solution is \(MMP_h = P_\zeta * (1 + \frac{1}{\epsilon_\zeta} \cdot \frac{\partial \zeta(h)}{\partial h})\), where \(P_\zeta\) is a
Labor turnover also impairs product quality. Malabre (1994), 126, argues that unions prevent publication of vacancy rates, so the union effect on turnover can't be measured. Union concealment thus abandons publicity as an ally, not always a sensible choice.

Dependence of quality on product development may also involve unions. Racial profiling, systematic undue accusation of criminal behavior, lowers product quality by reducing a victim's labor supply. Competitive firms that replace the "profiled" laborers by those who aren't without cost won't act against profiling.

Unions may internalize "profiling" costs. If highway police unjustly charge African Americans for drug or alcohol offenses, unions may demand new devices for cars. Low-cost breath analyzers might provide the accused evidence in court. Sensing devices in cars may save accident victims on insurance and repair costs.

Product development is a management prerogative, but as technology costs fall and discrimination costs rise, unions may act.\textsuperscript{16}

VI. Conclusion

A simple analysis provides definite answers on the effect of unions in standard cases. This analysis has been discussed and extended to cover differing market structures. Some spillover effects are also noted. As this analysis is pertinent for any input supplier who has market power, it has significant generality. Employers may offer "limit wages" (à la limit pricing) to deter unions, so the analysis may also apply to non-unionized labor markets.

References

Arrow, Kenneth, "The Economic Implications of Learning By Doing", quality price premium, and $e_x$ is its elasticity of demand. If some consumers buy items with defects, or $p_h \to -\infty$ and $p_h \neq 0$, monopoly quality is worse than competitive quality.

Deming's claim may hold for cases of industry technological change, or cases of multiple technique. Suppose there are two techniques, Y_1 and Y_2. Suppose there are also two firms, and they can't collude. Technique Y_1 has low fixed cost and produces some defects. Technique Y_2 produces fewer defects, but has high fixed cost. If fixed costs are just high enough, Cournot duopolists can't operate Y_2 profitably, but a monopoly can. (Cutting output price worsens duopoly profits with Y_2.)

\textsuperscript{16}Cadillac's Ultrasonic Rear Parking Assist\textsuperscript{TM} (http://www.cadillac.com/tech/index.htm), standard on 2002 Escalade models, suggests that such technology is not far off.


Mansfield, Edwin and Gary Yohe, Microeconomics: Theory /


